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OPTIMIZED PACKING HYPERSPHERES INTO A HYPERSPHERE WITH PROHIBITED ZONES

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PROBLEM STATEMENT

Let a collection of n hyperspheres

$$S_i(u_i) = \{X = (x_1, x_2, ..., x_d) \in \mathbf{R}^d : \|X - u_i\|^2 \le r_i^2\}, i \in I_n = \{1, 2, ..., n\}$$

r_i – radius of $S_i(u_i)$,

$u_i = (x_{i1}, x_{i2}, ..., x_{id})$ – vector of variable centers of $S_i(u_i)$, $i \in I_n$.

$C(r) = S(r) \setminus \text{int}(\bigcup_{l \in I_p} P_l)$ – placement domain

$S(r)=\{X \in \mathbf{R}^d : \|X\|^2 \le r^2\}$ – hyperspherical container,
r – variable radius,
$P_l = \{X \in \mathbf{R}^d : \|X - u_l^{inh}\|^2 \le (r_l^{inh})^2\}$ – hyperspherical prohibited zone, $l \in I_p = \{1, 2, ..., n_p\}$.

Problem. The hyperspheres $S_i(u_i)$, $i \in I_n$, have to be packed inside $S(r)$ providing restrictions on the prohibited zones P_l, $l \in I_p$, and the minimum allowable distances ρ_{ij} between hyperspheres $S_i(u_i)$ and $S_j(u_j)$, $i < j \in I_n$, so that the sizes of the container will be minimized.

MATHEMATICAL MODEL

$$\min_{(u,r)\in W\subset \mathbf{R}^{nd+1}} \kappa(u,r), \tag{1}$$

$$W=\{(u,r)\in \mathbf{R}^{nd+1}\colon \widehat{\Phi}_{ij}(u_i,u_j)\geq 0,\ i>j\in I,\ \Phi_i(u_i,r)\geq 0,\ i\in I_n\}, \tag{2}$$

$\kappa(u,r)=r$ – objective function,

$u=(u_1,u_2,...,u_n)$ – placement parameters,

$\widehat{\Phi}_{ij}(u_i,u_j)$ is the adjusted phi-function of the hyperspheres $S_i(u_i)$ and $S_i(u_i)$,

$\Phi_i(u_i,r)$ is the phi-function of the hypersphere $S_i(u_i)$ and the object $C^*(r)=\mathbf{R}^{d+1}\setminus \operatorname{int} C(r)$.

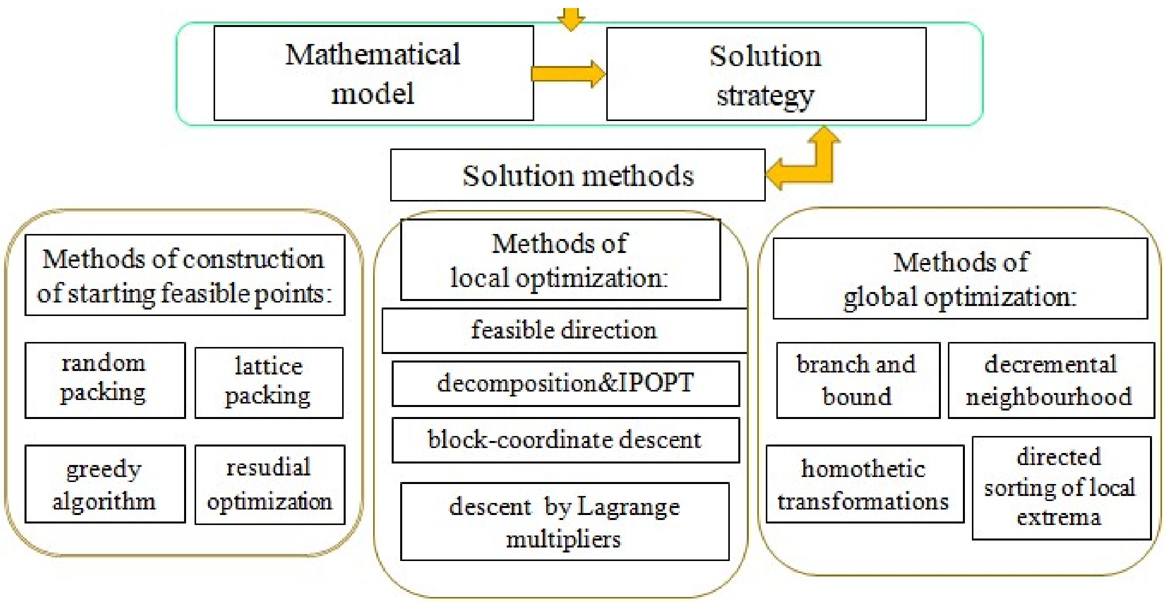
Main features:

- number of variables of the problem is $nd+1$ and of inequalities is $n+C_n^2$;
- Jacobian and Hessian matrices of the constraints of the problem are highly sparse;
- minimum of the linear objective function is found at the extreme points of W.

Extreme points are defined by systems of $nd+1$ equations specifying $\operatorname{fr} W$.

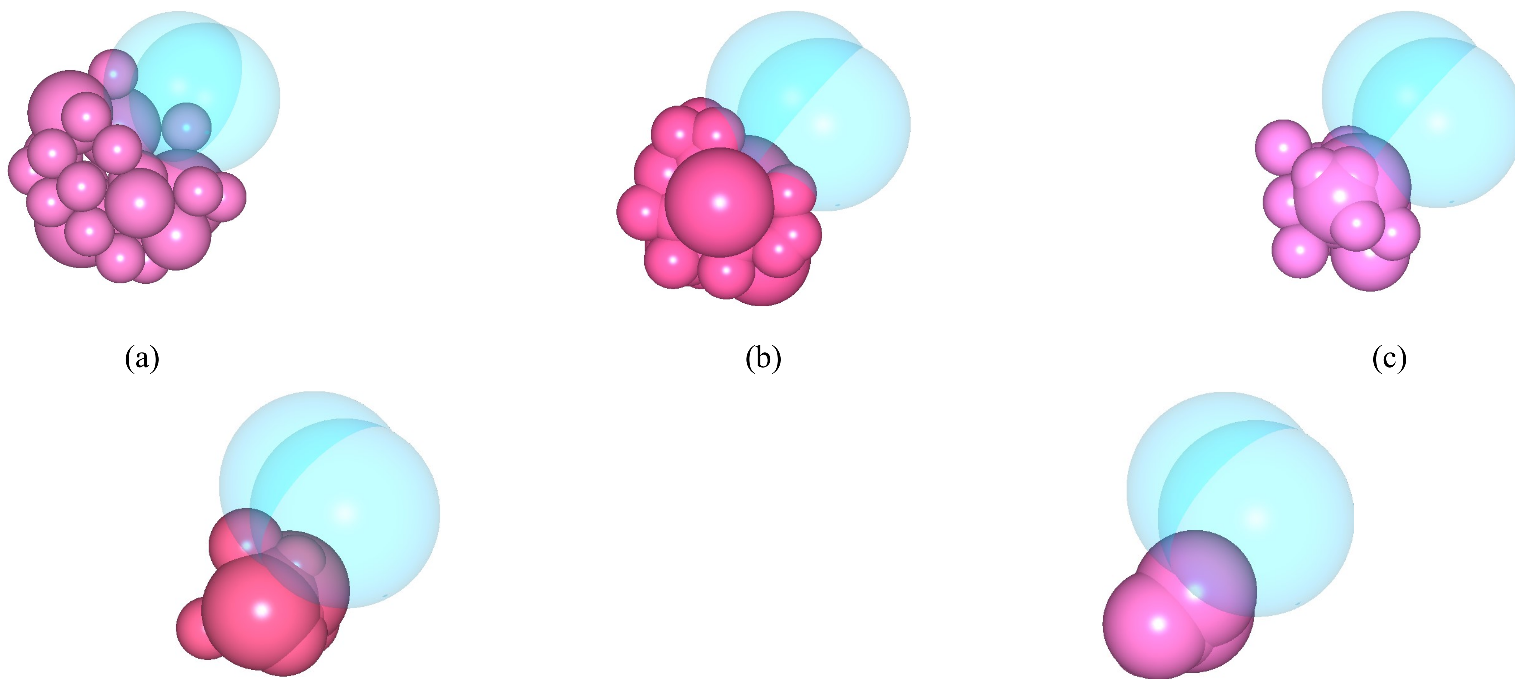
SOLUTION STRATEGY

1. The radii are supposed to be variable.
2. Feasible starting points are constructed based on the **random starting point algorithm** and the **residual optimization method**.
3. The corresponding local minima are found by the **decomposition method** [16] combined with **IPOPT**.
4. Using the auxiliary NLP subproblems, hyperspheres radii are redistributed filling up the container volume.



Numerical examples

Example 1. $n = 23$, $d = 3, 4, 8, 16, 24$, $\{r_i, i = 1, ..., 23\} = \{$1.2, 1.4, 1.7, 1.9 , 1.2, 1.4, 1.7, 1.9, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0$\}$ are radii of $S_i(u_i)$. The prohibited zone composed by two hyperspheres of radius $r = 3$ centered at $(2,3,2,1,...,1,1)$ and $(3,2,2,1,...,1,1)$ is given. The computational accuracy is 10^{-6}. Table 4 presents the rounded values of the objective function. CPU is limited by 5 minutes.



(a) (b) (c) (d) (e)

Optimized packings of hypespheres into a hypersphere with prohibited zones ($Ox_1x_2x_3$ projections): (a) d=3, (b) d=4, (c) d=8, (d) d=16, (e) d=24.

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